K-1668

M. A./M.Sc. (Final) Examination – 2025

MATHEMATICS

Paper-VII

(Optional)

ORTHOGONAL POLYNOMIALS

(Only for Due/Ex Students)

Time allowed : **Three Hours**

Maximum Marks : **100**

*Note : This question paper contains **three** sections as under :*

SECTION - A

1 (a) Define orthogonal polynomials.

(b) Write Bessel's inequality.

(c) Define Legendre's polynomial.

(d) Write generating function for Legendre's polynomials.

(e) Write the generating function for Hermite polynomials.

(f) Write rodrigues formula for $Hn(x)$.

(g) Write orthogonal property of simple Laguerre polynomials.

(h) Define associated Laguerre's polynomials.

(i) Define orthogonality of Jacobi polynomials.

(j) Define Gegenbauer polynomials $\alpha_n^{\lambda}(x)$.

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SECTION - B

UNIT - I

2 Show that the set of functions $\{\cos nx\}$, $n = 0,1,2,3,\ldots\ldots$ is orthogonal on the interval $-\pi \le x \le \pi$ and find the corresponding orthonormal set of functions.

OR

Let $m < n$ between any two zeroes of $P_m(x)$ there is a zero of $P_n(x)$. Show it.

UNIT - II

3 Express $P(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials.

OR

Prove that

$$\frac{1 - z^2}{(1 - 2xz + z^2)^{3/2}} = \sum_{n=0}^{\infty} (2n+1) P_n(x) z^n .$$

[Contd...

UNIT - III

4 Prove that $e^{2xt - t^2} = \sum_{n=0}^{\infty} H_n(x) \frac{t^n}{n!}$.

OR

Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}(e^{-x^2})$.

UNIT - IV

5 For Laguerre polynomials, Show that

$$(n+1)L_{n+1}(x) = (2n+1-x)L_n(x) - nL_{n-1}(x).$$

OR

$$x \dot{L}_n 1(x) = nLn(x) - nL_{n-1}(x).$$

[Contd...

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UNIT - V

6 For Jacobi Polynomials, Prove that

$$D^n P_n^{(\alpha,\beta)}(x) = \frac{1}{2^n}\frac{(1+\alpha+\beta)_{2n}}{(1+\alpha+\beta)_n}.$$

OR

$$DP_n^{(\alpha,\beta)}(x) = \frac{1}{2}(1+\alpha+\beta+n)\,P_{n-1}^{(\alpha+1,\beta+1)}(x).$$

SECTION - C

7 Prove that

$$\int_{-1}^{1}(x^2-1)P_{n+1}(x)P_n^{|}(x)dx = \frac{2n(n+1)}{(2n+1)(2n+3)}.$$

8 Prove for Hermite Polynomials,

$$H_n(x) = 2^n\left\{\exp.\left(-\frac{1}{4}\frac{d}{dx^2}\right)\right\}x^n.$$

9 Prove that for Laguerre's Polynomials,

$$\int_0^\infty e^{-x}L_n(x)L_m(x)dx = \begin{cases}0 & , m \neq n\\ 1 & , m = n\end{cases}$$

10 Show that

$$OF_1\left[\begin{matrix}-\\ \nu+\frac{1}{2}\end{matrix}\Big/\frac{t(x-1)}{2}\right]OF_1\left[\begin{matrix}-\\ \nu+1/2\end{matrix}\Big/\frac{t(x+1)}{2}\right] = \sum_{n=0}^{\infty}\frac{C_n^{\nu}(x)t^n}{(2\nu)_n\left(\nu+\frac{1}{2}\right)_n}$$

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