KBB-1657

M. A. / M. Sc. (Final) Examination, 2025

MATHEMATICS

PAPER - VII

TOPOLOGY

(Only for Due/Ex Students)

Time allowed : **Three Hours**

Max. Marks : **100**

This question paper contains three sections as under:

Section-A **खण्ड–अ** ***Max. Marks-10***

*This section contains **one compulsory** question with 10 parts, having 2 parts from each unit short answer in 20 words for each part. All questions carry **equal** marks.*

*इस खण्ड में एक अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई से 02 लघु प्रश्न लेते हुये कुल 10 लघु प्रश्न होंगे । प्रत्येक लघु प्रश्न का उत्तर 20 शब्दों से अधिक में न हो। सभी प्रश्नों के अंक **समान** हैं।*

Section-B **खण्ड–ब** ***Max. Marks-50***

*This section contains **10** questions having 2 questions from each unit. Answer 5 questions (250 words each) selecting one question from each unit. All questions carry **equal** marks.*

*इस खण्ड में प्रत्येक इकाई से 02 प्रश्न लेते हुये कुल 10 प्रश्न है । प्रत्येक इकाई से एक प्रश्न का चयन करते हुये, कुल 05 प्रश्नों के उत्तर देने हैं । प्रत्येक प्रश्न का उत्तर 250 शब्दों से अधिक में न हो। सभी प्रश्नों के अंक **समान** हैं।*

Section-C **खण्ड–स** ***Max. Marks-40***

*This section contains **04** descriptive type questions (questions may have sub-division). covering all units but not more than one question from each unit. Answer any two questions. (500 words each). All questions carry **equal** marks.*

*इस खण्ड में 04 प्रश्न वर्णनात्मक होंगे । प्रश्न में भाग भी हो सकते हैं जो सभी इकाइयों में से दिये जायेंगे, किन्तु एक इकाई में से एक से अधिक प्रश्न नहीं होगा । कोई दो प्रश्नों के उत्तर दिये जाने हैं । प्रत्येक प्रश्न का उत्तर 500 शब्दों से अधिक न हो। सभी प्रश्नों के अंक **समान** हैं।*

https://pyqlet.com

SECTION - A

1 (i) If $X = \{a, b, c\}$ and $T = \{\phi, \{a\}, \{b, c\}, X\}$ is a topology for X, then find closed sets in X.

(ii) Define second countable space.

(iii) Let (X, D) be the discrete topological space and (Y, V) be any topological space. Prove that every function $f : X \to Y$ is $D-V$ continuous.

(iv) Define Hausdorff space.

(v) Define Compact set.

(vi) Write the statement of Heine-Borel theorem.

(vii) Show that every indiscrete topological space (X, I), where X is non-empty set, is connected.

(viii) Define Locally connected space.

(ix) What is the Moore-Smith sequence?

(x) Define Discrete filter.

SECTION - B

UNIT - I

2 If T_1 and T_2 be two topology on a non-empty set X, then show that $T_1 \cap T_2$ is also a topology on X.

OR

3 Let (X, T) be a topological space. Prove that a sub-collection $\mathfrak{B}$ of T is base for T if and only if every T-open set can be expressed as the union of members of $\mathfrak{B}$.

UNIT - II

4 Prove that a function $f : (X, T) \to (Y, V)$ is continuous on X if and only if the inverse image of every V-open set is T-open.

OR

5 Prove that a topological space (X, T) is a T_1-space if and only if every singleton subset $\{x\}$ of X is T-closed.

https://www.uokonline.com

UNIT - III

6 Prove that the closed subsets of compact sets are compact.

OR

7 Prove that every sequentially compact metric space (X, d) is totally bounded.

UNIT - IV

8 Prove that a topological space X is disconnected if and only if there exists a non-empty proper subset of X which is both open and closed in X.

OR

9 Prove that the product space of two Hausdorff space is Hausdorff space.

UNIT - V

10 Let (X, D) be any discrete topological space. Show that a net $(f, X, A, \geq)$ in X converges to a point $x_0 \in X$ iff f is eventually in $\{x_0\}$, that is iff $\exists\ a_0 \in A$ s.t. $f_a = x_0, \forall a \geq a_0$ in A.

OR

11 Let $\{F_\lambda : \lambda \in \wedge\}$ be any non-empty family of filters on a non-empty set X. Prove that $F = \cap\{F_\lambda : \lambda \in \wedge\}$ is a filter on X.

SECTION - C

12 (a) Let $X = \{a, b, c, d, e\}$ and $T = \{\phi, \{b\}, \{c, d\}, \{b, c, d\}, \{a, b, c, d\}, x\}$

Find (i) interior, (ii) exterior, (iii) frontier of the subset $A = \{c\}$ of X.

(b) Let U consists of ϕ and all those subsets G of R having the property that two each $x \in G$, these exists $\in > 0$ such that $(x - \in, x + \in) \subset G$. Prove that U is a topology for R.

13 (a) Let $X = \{1, 2, 3, 4\}$ and let $T = \{X, \phi, \{1\}, \{2\}, \{1, 2\}, \{2, 3, 4\}\}$ be a topology defined on X. If a function $f : X \to X$ is defined by $f = \{(1, 2), (2, 4), (3, 2), (4, 3)\}$

Show that (i) f is not continuous at 3 bur (ii) f is continuous at 4.

(b) Show that every T_2-space is a T_1-space, but the converse need not be true.

14 Prove that a metric space X is compact if and only if it is sequentially compact.

15 (a) Show that the space (R, S) is disconnected, where S is the lower limit topology on R.

(b) A topological space X is disconnected iff there exists a continuous mapping of X onto the discrete two point space $\{0,1\}$.

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