QZ-1657

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**M. A. / M. Sc. (Final) Examination, 2024**

**MATHEMATICS**

**PAPER - VII**

**TOPOLOGY**

Time Allowed : **Three Hours**

Maximum Marks : **100**

*This question paper contains three sections as under:*



## SECTION-A

***Max. Marks - 10***

*This section contains **one compulsory** question with **10** parts, having 2 parts from each unit, short answer in **20** words for each part. All questions carry **equal** marks.*

## SECTION-B

***Max. Marks-50***

*This section contains **10** questions having 2 questions from each unit. Answer **5** questions (250 words each) selecting one question from each unit. All questions carry **equal** marks.*

## SECTION-C

***Max. Marks - 40***

*This section contains **04** descriptive type questions (questions may have sub-division) covering all units but not more than one question from each unit. Answer any **two** questions. (**500** words each). All questions carry **equal** marks.*



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## SECTION - A

1 (i) Define Co-countable topology.

(ii) What is the second countable space ?

(iii) Define topological property.

(iv) What is the regular space ?

(v) What is the Bolzano - Weierstrass property ?

(vi) Define $\in$-net.

(vii) Let $(X, T)$ be a topological space, where $X = \{a,b,c\}$ and $T = \{\phi \ \{a\}, \{b,c\}, X\}$ Show that the space $(X, T)$ is disconnected.

(viii) Define Tychonoff topology.

(ix) Define cluster point of a net.

(x) Let $X = \{a, b, c, d\}$ and $C = \{\{a, b\}, \{b, c\}\}$ be a non-empty family of subsets of $X$. Is family $C$ generates filter of $X$, if so find filter generated by $C$.



## SECTION - B

### UNIT - I

2 Let $X = \{a, b, c, d, e\}$ and $T = \{(\phi) \{b\}, \{d, e\} \{b, d, e\}, \{a, c, d, e\}, X\}$ Find all limit points of the set $A = \{b, c, d\}$.

3 Prove that every second countable space is first countable.

### UNIT - II

4 Let $(X, T)$ and $(Y, V)$ be two topological spaces and let $f$ be a one-one onto mapping of $X$ onto $Y$. Prove that $f$ is open and continuous if and only if $f$ is a homeomorphism.

5 Let $(X, T)$ be a topological space. Prove that $T$ is a Hausdorff topology for $X$ if and only if the intersection of all closed nbd's of each point of $X$ is a singleton.



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## UNIT - III

6 Prove that every compact subset $A$ of a Hausdorff space $X$ is closed.

7 Let $A$ be a compact subset of a matrix space $(X, d)$. Show that for every $B \subset X$, there exists a point $x_0 \in A$ such that $d(x_0, B) = d(A, B)$.

## UNIT - IV

8 Let $(X, T)$ be a topological space and let $E$ be a connected subset of $X$ such that $E \subset A \cup B$, where $A$ and $B$ are separable sets. Prove that $E \subset A$ or $E \subset B$.

9 Let $\{(X_\lambda, T_\lambda) : \lambda \in \wedge\}$ be a collection of topological spaces and let $X = X\{X_\lambda ; \lambda \in \wedge\}$. Prove that $X$ is compact relative to the product topology if and only if each space $X_\lambda$ is compact.



## UNIT - V

10 Let $(X, T)$ be a topological space and let $(f, X, A, \geq)$ be a net in $X$. For each $a \in A$ let $M_a = \{f(x) : x \geq a \text{ in } A\}$. Prove that a point $p$ of $X$ is a cluster point of $f$ if and only if $p \in \bar{M}_a$ and $\forall a \in A$.

11 Let $F$ be a filter on a non-empty set $X$ and $A \subset X$. Prove that there exits a fileter $F'$ finer than $F$ such that $A \in F'$ if and only if $A \cap B \neq \phi$, $\forall B \in F$.

## SECTION - C

12 (a) Let $(X, T)$ be a topological space and let $A$ be a subset of $X$. Prove that $A^\circ$ is the set of all those points of $A$ which are not limit point of $A'$.

(b) Let $(X, T)$ be a topological space and $B \subset T$ be a basis for $T$. Prove that $A$ is open if and only if for each $x \in A$, $\exists \cup \in B$ such that $x \in \cup \subset A$.



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13 State and prove Urysohn's lemma.

14 (a) Prove that a metric space $X$ is compact if and only if it is sequentially compact.

(b) State and prove Heine Borel theorem.

15 Let $(X, T)$ and $(Y, V)$ be two topological spaces and let $(X \times Y, W)$ be the product space of two spaces. Prove that the projection $\Pi_x, \Pi_y$ are cotinuous and open mappings.
Also, prove that the product topology $W$ is the coarsest topology for which the projections are continuous.

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