This question paper contains 7 printed pages]

1657

M.A./M.Sc. (Final) Examination, 2022

MATHEMATICS

Paper VII (Comp.)

(Topology)

Time Allowed : Three Hours

Maximum Marks : 100

This question paper contains ***three*** *sections as under :*

Section-A **खण्ड 'अ'** ***Max. Marks-10***

This section contains ***one*** *compulsory question with* ***10*** *parts, having* ***2*** *parts from each unit, short answer in* ***20*** *words for each part.* ***All*** *questions carry equal marks.*

इस खण्ड में ***एक*** *अनिवार्य प्रश्न है जिसमें प्रत्येक इकाई से* ***2*** *लघु प्रश्न लेते हुए कुल* ***10*** *लघु प्रश्न होंगे। प्रत्येक लघु प्रश्न का उत्तर* ***20*** *शब्दों से अधिक में न हो।* ***सभी*** *प्रश्नों के अंक समान हैं।*

Section-B **खण्ड 'ब'** ***Max. Marks-50***

This section contains ***10*** *questions having* ***2*** *questions from each unit. Answer* ***5*** *questions (****250*** *words each) selecting* ***one*** *question from each unit.* ***All*** *questions carry equal marks.*

इस खण्ड में प्रत्येक इकाई से ***2*** *प्रश्न लेते हुए कुल* ***10*** *प्रश्न हैं। प्रत्येक इकाई से* ***एक*** *प्रश्न का चयन करते हुए, कुल* ***5*** *प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न का उत्तर* ***250*** *शब्दों से अधिक न हो।* ***सभी*** *प्रश्नों के अंक समान हैं।*

Section-C **खण्ड 'स'** ***Max. Marks-40***

This section contains ***4*** *descriptive type questions (questions may have sub-divisions) covering all units but not more than one question from each unit. Answer any* ***two*** *questions (****500*** *words each).* ***All*** *questions carry equal marks.*

इस खण्ड में ***4*** *वर्णनात्मक प्रश्न होंगे (प्रश्नों में उप-भाग भी हो सकते हैं) जो सभी इकाइयों में से दिये जायेंगे, किन्तु एक इकाई से एक से अधिक प्रश्न नहीं होगा। किन्हीं* ***दो*** *प्रश्नों के उत्तर दिये जाने हैं। प्रत्येक प्रश्न का उत्तर* ***500*** *शब्दों से अधिक न हो।* ***सभी*** *प्रश्नों के अंक समान हैं।*

Section A/खण्ड-अ

1. (*i*) Define neighbourhood of a point in a topological space.

(*ii*) Explain the closure of a set.

(*iii*) Define homeomorphism.

(*iv*) What is Hausdorff space ?

(*v*) Define locally compact space.

(*vi*) Explain briefly the equivalence of compactness.

(*vii*) Define connected space.

(*viii*) State Tychonoff theorem.

(*ix*) Define the subnet.

(*x*) Explain the convergence of filters.

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Section B

UNIT-I

2. Let A be a subset of the topological space X; let A' be the set of all limit points of A, then prove that :

$$\overline{A} = A \cup A'$$

Or

3. If X has a countable base for its topology, then prove that any open cover of X contains a countable subcover.

UNIT-II

4. Prove that a topological space (X, τ) is a T_1-space iff every subset of X consisting of exactly one point is closed.

Or

5. If $f : X \to Y$ be a bijection, then prove that the following properties of f are equivalent :

(*a*) f is a homomorphism

(*b*) f is continuous and open

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(c) f is continuous and closed

(d) $f(\overline{A}) = \overline{f(A)}$ for each $A \subset X$.

UNIT-III

6. Prove that the continuous image of a compact space is compact.

Or

7. Prove that every finite topological space is compact. Give *one* example.

UNIT-IV

8. Prove that the union of a collection of connected sets that have a point in common is connected.

Or

9. Prove that the finite product of locally connected space is locally connected.

UNIT-V

10. Prove that a topological space is Hausdorff iff limits of all nets in it are unique.

Or

11. Let S be a family of subsets of a Set X. Then prove that there exists a filter on X having S as a sub-base if and only if S has the finite intersection property.

Section C

12. Let (X, τ_1) and (Y, τ_2) be two topological spaces and $f : X \to Y$ be a mapping.

Then show that the following are equivalent :

(a) f is continuous

(b) for every closed set B in Y the set $f^{-1}(B)$ is closed in X.

(c) for every subset of X one has $f(\overline{A}) \subset \overline{f(A)}$.

13. Show that a subset of R is connected if and only if it is an interval.

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14. Prove that the product space (X, Y, W) is compact if and only if each of the space (X, τ) and (Y, V) is compact.

15. Prove that a topological space (X, τ) is a T_1 space if and only if every singleton subset $\{x\}$ of X is T-closed.

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